Pearson Edexcel

Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Applied) (Public release version)

Resource Set 1: Topic 7

Kinematics (Test 3)

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. [*In this question the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upward.*]

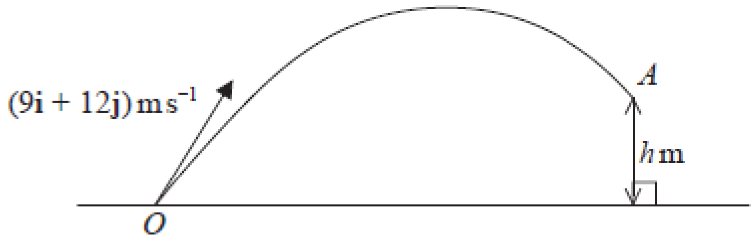


Figure 2

A small ball is projected from the fixed point O on horizontal ground with velocity $(9\mathbf{i} + 12\mathbf{j})$ m s^{-1}. The ball passes through the point A which is h metres vertically above the level of O, as shown in Figure 2. The velocity of the ball at the instant it passes through the point A is $\lambda(\mathbf{i} - \mathbf{j})$ m s^{-1}, where λ is a positive constant. The ball is modelled as a particle moving freely under gravity.

(a) Find the value of h.

(4)

a) $\underline{v} = \underline{u} + \underline{a}t$

$\begin{pmatrix} \lambda \\ -\lambda \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t$

$\lambda = 9$

$-9 = 12 - 9.8t \Rightarrow t = \frac{-21}{-9.8} = 2.14\text{s}$

$S = ut + \frac{1}{2}at^2$

$h = 12(2.14) - 4.9(2.14)^2$

$= 3.21\text{m}$

(b) State the minimum speed of the ball as it moves from O to A. **(1)**

b) 9ms^{-1}

The horizontal component is constant but the vertical component is 0 at the vertex

(c) Find the length of time for which the speed of the ball is less than 12 m s^{-1}. **(4)**

c) Speed $= |v| = \sqrt{v_x^2 + v_y^2}$

$\sqrt{9^2 + v_y^2} < 12$

$9^2 + v_y^2 < 144$

$v_y^2 < 63$

$|v_y| < 3\sqrt{7} \implies -3.7 < v_y < 3.7$

$v = u + at$ | $v = u + at$

$u + at < 3\sqrt{7}$ | $u + at > -3\sqrt{7}$

$12 - 9.9t < 3\sqrt{7}$ | $12 - 9.8t > -3\sqrt{7}$

$\implies t > 0.41s$ | $\implies t < 2.03s$

$2.03 - 0.41 = \underline{1.62s}$

The model could be refined by considering air resistance.

(d) Suggest one other refinement to the model that would make it more realistic.

(1)

d) The value of g is not constant at different heights so that could be taken into account

(Total for Question 1 is 10 marks)

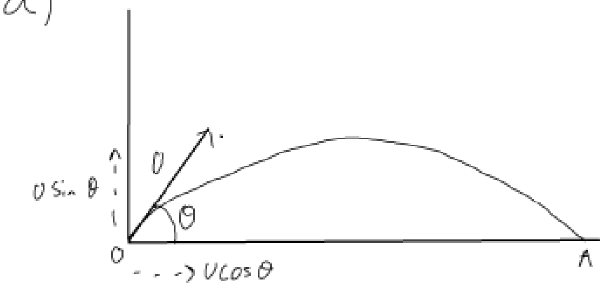
2. A small ball is projected with speed u from a point O on horizontal ground. The angle of projection is θ to the horizontal, where $0 < \theta < 90°$. The ball hits the ground at the point A.

The ball is modelled as a particle moving freely under gravity.

(a) Show that, according to the model, $OA = \dfrac{u^2 \sin 2\theta}{g}$.

(5)

a)



$V = U + at$

$0 = U\sin\theta - gt \Rightarrow t = \dfrac{U\sin\theta}{g}$ (time to vertex because $V=0$)

So $2t$ = time from O to A

Distance OA = Speed x time

$= U\cos\theta \times \dfrac{2U\sin\theta}{g} = \dfrac{2U^2 \sin\theta\cos\theta}{g} = \dfrac{U^2(2\sin\theta\cos\theta)}{g}$

Identity: $\sin 2A = 2\sin A\cos A$

$OA = \dfrac{U^2 \sin 2\theta}{g}$

A golfer hits a golf ball with speed 25 m s^{-1} from a point X on horizontal ground.

The golf ball hits the ground at the point Y. The angle of projection is θ to the horizontal, where $0 < \theta < 90°$. The golfer requires the distance XY to be at least 40 m.

The golf ball is modelled as a particle moving freely under gravity.

(b) Find, according to the model, the size of the largest possible angle θ.

(2)

b) $OA = \frac{u^2 \sin 2\theta}{g}$

$40 > \frac{25^2 \sin 2\theta}{g}$

$\frac{40g}{625} > \sin 2\theta$

$\frac{1}{2}\sin^{-1}\left(\frac{40(9.8)}{625}\right) > \theta$

$19.4° > \theta$

Given that $\theta = 30°$ and that the golf ball is more than 3 m above the ground for T seconds,

(c) find the value of T.

c) $s = ut + \frac{1}{2}at^2$

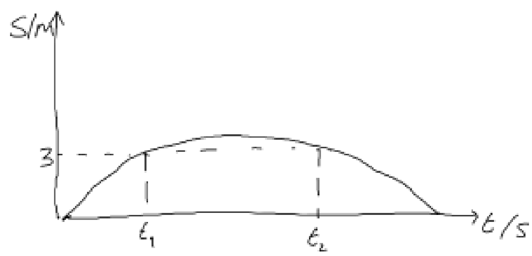
$3 = t(u\sin 30) - 4.9t^2$

$\Rightarrow 4.9t^2 - 12.5t + 3 = 0$

$\Rightarrow t = 2.28s \quad t = 0.28s$

$T = 2.28 - 0.28 = 2s$

$T = 2s$



(4)

(Total for Question 2 is 11 marks)

3.

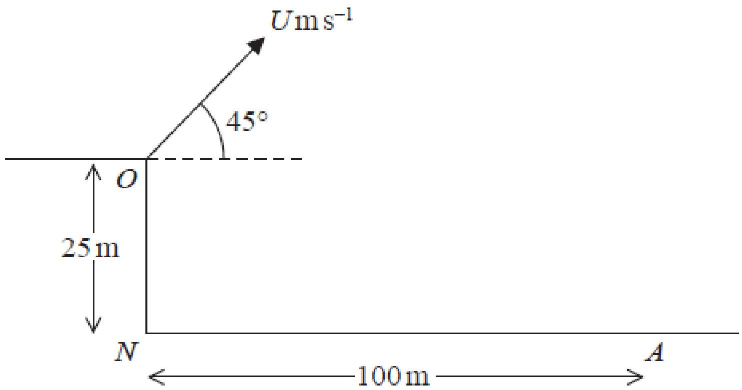


Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$ **(6)**

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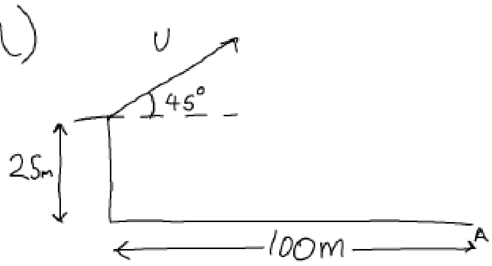
a)



$$S = Ut + \frac{1}{2}at^2$$

$$-25 = (U\sin 45)t - 4.9t^2$$

$$= U\sin 45\left(\frac{100}{U\cos 45}\right) - 4.9\left(\frac{100}{U\cos 45}\right)^2$$

$$= 100\tan 45 - \frac{4.9(100)^2}{U^2\cos^2 45}$$

$$-25 - 100\tan 45 = \frac{-4.9(100)^2}{U^2\cos^2 45}$$

$$U = \sqrt{\frac{-4.9(100)^2}{(-25-100\tan 45)\cos^2 45}} = 28$$

$$100 = (U\cos 45)t$$

$$\Rightarrow t = \frac{100}{U\cos 45}$$

Considering the distance OA and the horizontal component of velocity

(b) find the greatest height of the ball above the horizontal ground *NA*. **(3)**

b) $V = U + at$

$0 = 28 \sin 45 - gt$

$\Rightarrow t = \frac{28 \sin 45}{g}$

$S = ut + \frac{1}{2} at^2$

$= 28 \sin 45 \left(\frac{28 \sin 45}{g}\right) - 4.9 \left(\frac{28 \sin 45}{g}\right)^2$

$= 20m$

$20 + 25 = 45$

45m above the ground

In a refinement to the model of the motion of the ball from *O* to *A*, the effect of air resistance is included.

This refined model is used to find a new value of *U*.

(c) How would this new value of *U* compare with 28, the value given in part (a)? **(1)**

c) This new U is larger than 28 to counteract the resistive force of air resistance

(e) State one further refinement to the model that would make the model more realistic.

d) Model g as variable not constant

(1)

(Total for Question 3 is 11 marks)

4. [In this question use $g = 10 \text{ m s}^{-2}$.]

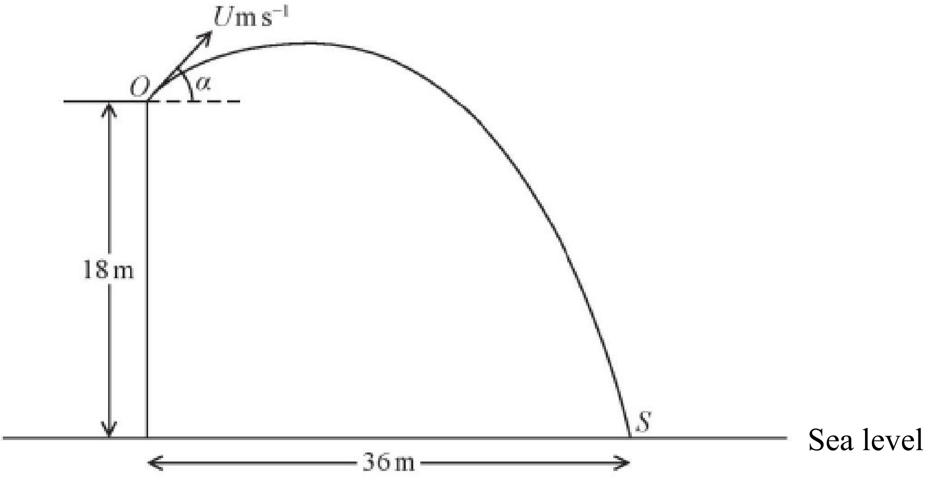


Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff. The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \dfrac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.

The stone is modelled as a particle moving freely under gravity with $g = 10 \text{ ms}^{-2}$

Find

(a) the value of U,

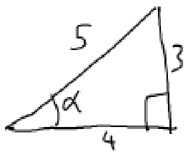
(6)

a) $36 = (U\cos\alpha)t$

$\Rightarrow t = \dfrac{36}{U\cos\alpha} = \dfrac{36}{U(\frac{4}{5})}$

$t = \dfrac{45}{U}$

$\tan\alpha = \dfrac{3}{4}$



So $\sin\alpha = \dfrac{3}{5}$, $\cos\alpha = \dfrac{4}{5}$

$S = ut + \frac{1}{2}at^2$

$-18 = t(U\sin\alpha) - \dfrac{g}{2}t^2$

$= \dfrac{45}{U}\left(U\left(\frac{3}{5}\right)\right) - \dfrac{g}{2}\left(\dfrac{45}{U}\right)^2$

$= 27 - \dfrac{g(45^2)}{2U^2}$

$-45 = \dfrac{-g(45)^2}{2U^2}$

$U = \sqrt{\dfrac{-g(45)^2}{-45}} = 15$

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures. **(5)**

b) $S = ut + \frac{1}{2}at^2$

$-7.2 = (U\sin\alpha)t - 5t^2$

$\Rightarrow 5t^2 - 9t - 7.2 = 0$

$t = 2.4s, \ t = -0.6s$

$t > 0$ so $t = 2.4$

$V = U + at$

$= U\sin\alpha - g(2.4)$

$= 9 - 24$

$V = -15$

Speed $= |v| = \sqrt{15^2 + 9^2} = 17ms^{-1}$

(c) Suggest two improvements that could be made to the model.

c) Air resistance could be taken into account

Making g variable instead of constant

(2)

(Total for Question 4 is 13 marks)

5.

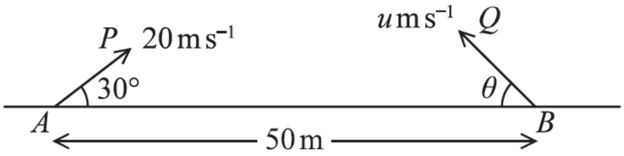


Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB.

Ball Q is projected from B with speed u m s^{-1} at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

a) $V = U + at$

$= 20\sin 30 - g(2)$ } Vertical component

$= -9.6$

The horizontal component remains constant at $20\cos 30$

$V = \begin{pmatrix} 10\sqrt{3} \\ -9.6 \end{pmatrix}$ ms^{-1}

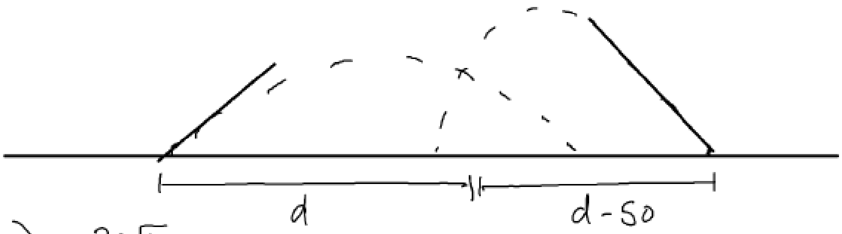
(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

b) (i)



$d = 2(20\cos 30) = 20\sqrt{3}$

$50 - d = 50 - 20\sqrt{3} = t(u\cos\theta)$ $(t = 2)$

$25 - 10\sqrt{3} = u\cos\theta$

They meet at the same height

$s = ut + \frac{1}{2}at^2$

$= 20 - 4.9(2)$

$= 0.4$

$0.4 = ut + \frac{1}{2}at^2$

$= 2u\sin\theta - 4.9(4)$

$20 = 2u\sin\theta$

$10 = u\sin\theta$

$\frac{u\sin\theta}{u\cos\theta} = \tan\theta = \frac{10}{25 - 10\sqrt{3}}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{10}{25 - 10\sqrt{3}}\right) = 52.5^\circ$

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

c) The fact that g is modelled as constant not variable

(1)

(Total for Question 5 is 13 marks)

6.

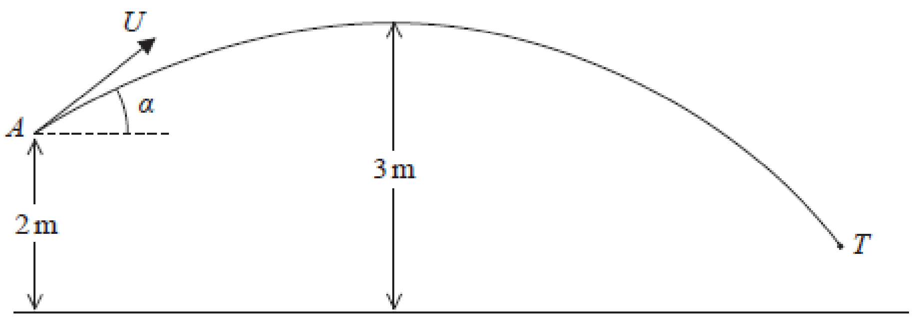


Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground. The target is modelled as being the point T, as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

a) $v^2 = u^2 + 2as$

$0 = (U \sin\alpha)^2 + 2(-g)(1)$ } At the vertex, the displacement is 1m and the velocity is 0ms⁻¹

$2g = U^2 \sin^2\alpha$

$U^2 = \dfrac{2g}{\sin^2\alpha}$

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α.

(9)

b) $20 = t(U\cos\alpha)$

$t = \frac{20}{U\cos\alpha}$

$-1.25 = (U\sin\alpha)t - \frac{g}{2}t^2$

$= U\sin\alpha\left(\frac{20}{U\cos\alpha}\right) - 4.9\left(\frac{20}{U\cos\alpha}\right)^2$

$= 20\tan\alpha - 4.9(20)^2\left(\frac{1}{U^2}\right)\left(\frac{1}{\cos^2\alpha}\right)$

$= 20\tan\alpha - 4.9(400)\left(\frac{\sin^2\alpha}{2g}\right)\left(\frac{1}{\cos^2\alpha}\right)$

$-1.25 = 20\tan\alpha - 100\tan^2\alpha$

$\Rightarrow$ Let $t = \tan\alpha$

$100t^2 - 20t - 1.25 = 0$

$t = \frac{1}{4} \Rightarrow \alpha = \arctan\frac{1}{4}$

(c) State one limitation of the model that could affect your answer to part (b). **(1)**

c) The model does not take into account air resistance

(d) Find the time taken for the ball to travel from A to T.

d) $t = \frac{20}{U\cos\alpha}$ $\quad U = \sqrt{\frac{2g}{\sin^2\alpha}} = \frac{\sqrt{2g}}{\sin\alpha}$

$= \frac{20\sin\alpha}{\sqrt{2g}\cos\alpha} = \frac{20}{\sqrt{2g}}\tan\alpha = \frac{20}{\sqrt{2g}}\left(\frac{1}{4}\right)$

$= \frac{5}{\sqrt{2g}} = 1.1\text{s}$

(3)

(Total for Question 6 is 15 marks)